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Fall Semester

Course of Power System Analysis

The per-unit method on transformers

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Outline

Recap

Equivalent circuit of a transformer

Per-unit quantities

Electrical quantities such as voltage, current, power or impedance are often expressed as a percentage or **per unit of a base or reference** value specified for each. For instance, if a base voltage of 120 kV is chosen, voltages of 108, 120, and 126 kV become 0.90, 1.00, and 1.05 per-unit.
The per-unit value of any quantity is defined as the ratio of the quantity to its base.

If the given quantity is expressed as a complex number $\bar{A} = A_1 + jA_2$, **the same base value is assumed for both the real part and the imaginary coefficient:**

$$\bar{a} = \frac{\bar{A}}{A_b} = \frac{A_1 + jA_2}{A_b} = \frac{A_1}{A_b} + j\frac{A_2}{A_b} = a_1 + ja_2$$

As a direct consequence, the **module of $\bar{A}$ divided by the base A_b is equal to the module of $\bar{a}$** as shown in the following equation:

$$\frac{|\bar{A}|}{A_b} = \frac{\sqrt{A_1^2 + A_2^2}}{A_b} = \sqrt{\frac{A_1^2}{A_b} + \frac{A_2^2}{A_b}} = \sqrt{a_1^2 + a_2^2} = |\bar{a}|$$

Per-unit quantities

The choice of the bases has to be **consistent.**
A system of base values is consistent if the per-unit value of a quantity, which depends on other quantities according to a "**physical law**", can be obtained from the per-unit values of these quantities using the above-mentioned "physical law".

In an electrical system the bases for voltages (V_b), **currents** (I_b), **powers** (A_b) **and impedances** (Z_b) **are so related that the selection of any two of them determines the base values of the remaining two.**

Example:
If we specify the base values of current and voltage, base impedance and base kVA can be determined as follow:

- The **base kVA** in single-phase systems is the product of base voltage in kilovolts and base current in amperes.
- The **base impedance** is that impedance which will have a voltage drop across it equal to the base voltage when the current flowing in the impedance is equal to the base value of the current

Per-unit impedances

- Manufacturers usually specify the impedance of a piece of apparatus in per-unit on the base of the **nameplate rating**.

- The per-unit impedances of machines of the same type and widely different rating usually lie within a narrow range. When the impedance is not known definitely, it is generally possible to select from tabulated average values a per-unit impedance which will be reasonably correct.

- Experience in working with per-unit values brings familiarity with the proper values of per-unit impedance for different types of apparatus.

- **The per-unit impedance, once expressed on the proper base, is the same referred to either side of any transformer (to be seen in the lecture on transformers)**

- **The way in which transformers are connected in three-phase circuits does not affect the per-unit impedances of the equivalent circuit** although the transformer connection does determine the relation between the voltage bases on the two sides of the transformer **(to be seen in the lecture on transformers).**

Outline

Recap

Equivalent circuit of a transformer

Equivalent circuit of a transformer

The equations ruling the single-phase equivalent circuit of a three-phase transformer shown in Fig. (1.1) defines the behaviour of the transformer as a two-port circuit (see lecture on transformers):

$$\bar{V}_1 = a\bar{V}_2 + \bar{Z}_{cc} \cdot \frac{\bar{I}_2}{a} \quad (1.13)$$

$$\bar{I}_1 = \bar{Y}_0(a\bar{V}_2) + (\bar{Y}_0\bar{Z}_{cc} + 1) \cdot \frac{\bar{I}_2}{a} \quad (1.14)$$

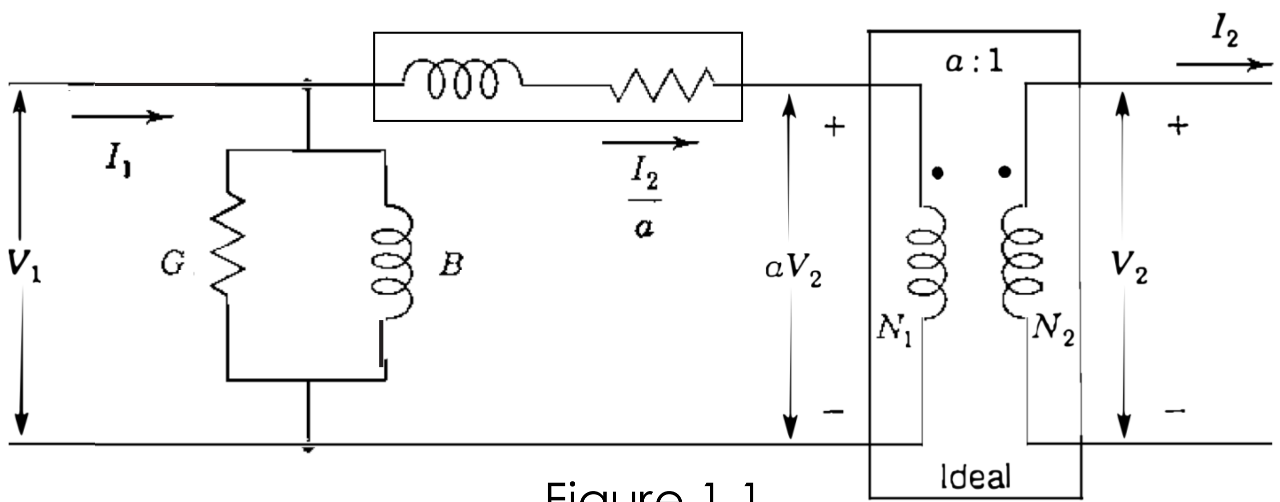


Figure 1.1
Single-phase equivalent circuit of a three-phase transformer

Equivalent circuit of a transformer

As shown in Slide 10, let's now consider the base values V_{b1}, V_{b2} equal to the line-to-line voltage for the two sides of the transformer and the base value A_b equal to the three-phase power. The parameters of the transformer Y_0 and Z_{cc}, expressed in per-unit, will be as follow:

$$z_{cc} = Z_{cc}\frac{A_b}{V_{b_1}^2} \qquad\qquad y_0 = Y_0\frac{V_{b_1}^2}{A_b}$$

It is now possible to write Eq.(1.13) in per-unit by multiplying it by $\frac{\sqrt{3}}{V_{b1}}$:

$$\bar{V}_1 \cdot \frac{\sqrt{3}}{V_{b_1}} = a\bar{V}_2 \cdot \frac{\sqrt{3}}{V_{b_1}} \cdot \frac{V_{b_2}}{V_{b_2}} + \bar{Z}_{cc} \cdot \frac{\bar{I}_2}{a} \cdot \frac{\sqrt{3}}{V_{b_1}} \qquad (1.13)$$

$$\bar{v}_1 = a\bar{v}_2\frac{V_{b_2}}{V_{b_1}} + \bar{z}_{cc}\frac{\bar{\imath}_2}{a}\frac{V_{b_1}}{V_{b_2}} \qquad (1.15)$$

Similarly, Eq. (1.14) can be transformed as follow:

$$\bar{I}_1 \cdot \frac{\sqrt{3}V_{b_1}}{A_b} = \bar{Y}_0(a\bar{V}_2) \cdot \frac{\sqrt{3}V_{b_1}}{A_b} \cdot \frac{V_{b_1}}{V_{b_1}} \cdot \frac{V_{b_2}}{V_{b_2}} + (\bar{Y}_0\bar{Z}_{cc} + 1) \cdot \frac{\bar{I}_2}{a} \cdot \frac{\sqrt{3}V_{b_1}}{A_b} \cdot \frac{V_{b_2}}{V_2} \qquad (1.14)$$

$$\bar{\imath}_1 = \bar{y}_0(a\bar{v}_2) \cdot \frac{V_{b_2}}{V_{b_1}} + (\bar{y}_0\bar{z}_{cc} + 1) \cdot \frac{\bar{\imath}_2}{a} \cdot \frac{V_{b_1}}{V_{b_2}} \qquad (1.16)$$

Equivalent circuit of a transformer

The obtained equations are the following:

$$\bar{v}_1 = a\bar{v}_2 \frac{V_{b_2}}{V_{b_1}} + \bar{z}_{cc} \frac{\bar{\imath}_2}{a} \frac{V_{b_1}}{V_{b_2}} \quad (1.15)$$

$$\bar{\imath}_1 = \bar{y}_0(a\bar{v}_2) \cdot \frac{V_{b_2}}{V_{b_1}} + (\bar{y}_0\bar{z}_{cc} + 1) \cdot \frac{\bar{\imath}_2}{a} \cdot \frac{V_{b_1}}{V_{b_2}} \quad (1.16)$$

If the choice of V_{b_1} and V_{b_2} has been made such that the turn ratio a is equal to V_{b_1}/V_{b_2}, than we can write:

$$\bar{v}_1 = \bar{v}_2 + \bar{z}_{cc}\bar{\imath}_2 \quad (1.17)$$

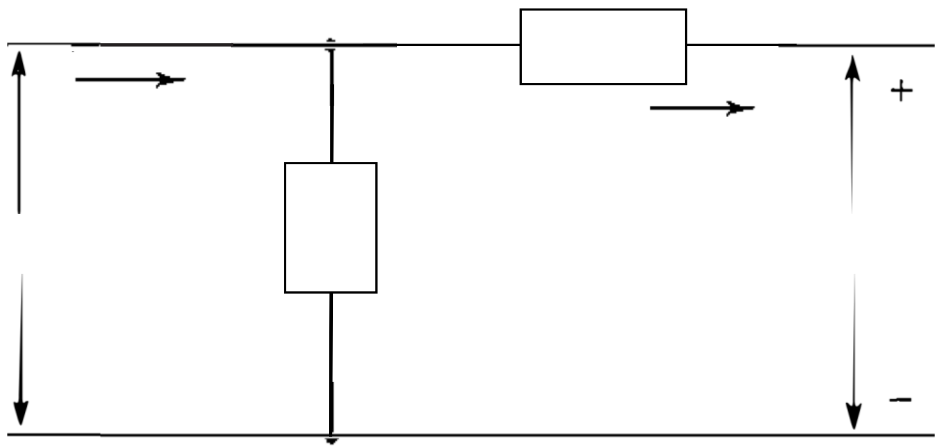
$$\bar{\imath}_1 = \bar{y}_0\bar{v}_2 + (\bar{y}_0\bar{z}_{cc} + 1)\,\bar{\imath}_2 \quad (1.18)$$

It is important to note that, in Eq.(1.14) and (1.15) **the turn ratio has disappeared.**

Equivalent circuit of a transformer

The matrix form of the previous equations is here expressed:

$$\begin{bmatrix} \bar{v}_1 \\ \bar{\imath}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \bar{v}_2 \\ \bar{\imath}_2 \end{bmatrix} = \begin{bmatrix} 1 & \bar{z}_{cc} \\ \bar{y}_0 & \bar{y}_0 \bar{z}_{cc} + 1 \end{bmatrix} \begin{bmatrix} \bar{v}_2 \\ \bar{\imath}_2 \end{bmatrix} \quad (1.19)$$



Figure(1.2)

Single-phase equivalent circuit of a three-phase transformer with a real transformation ratio r equal to the ratio between the basic values chosen for the primary and secondary voltage.

Equivalent circuit of a transformer
Summary

Two main steps:

- From Equivalent Circuit 1 (EC1) to EC2
  Hyp#1: Voltage drop on primary winding very small

- From EC2 to EC3
  Hyp#2: Per-unit method with $r = V_{b_1}/V_{b_2}$.

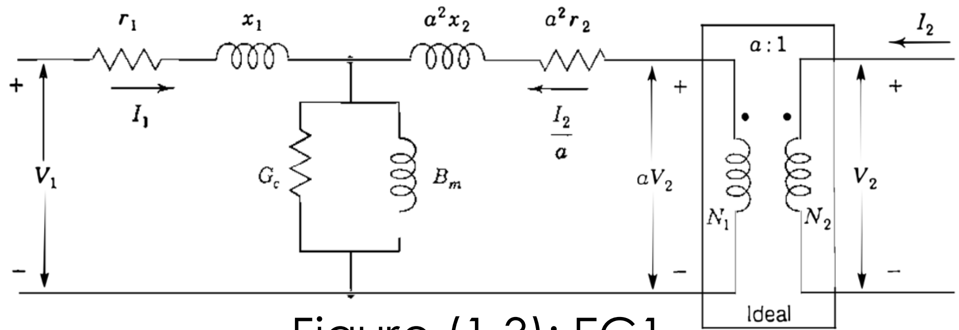


Figure (1.3): EC1

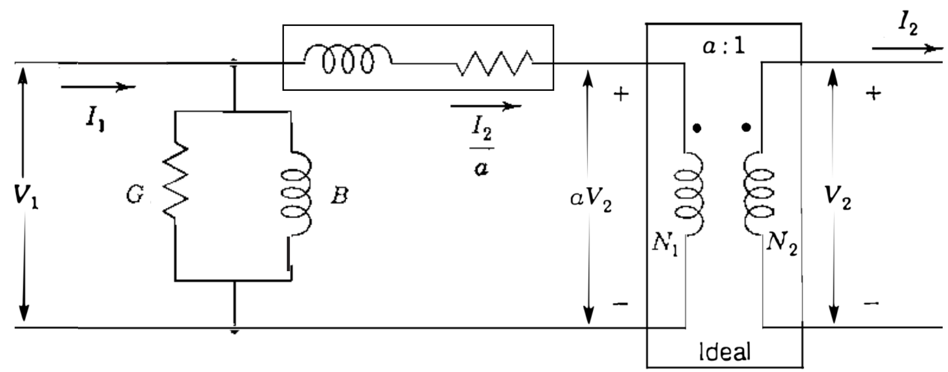
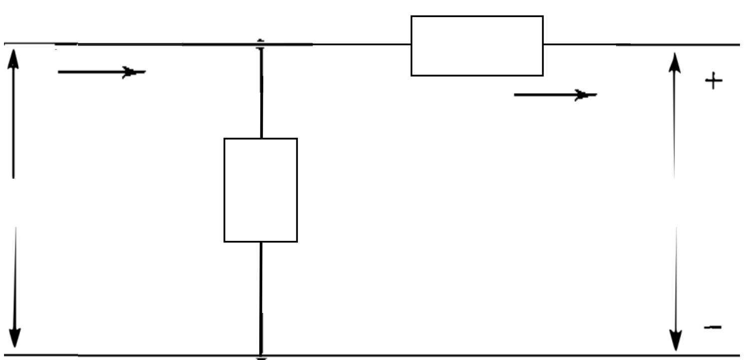


Figure (1.1): EC2

Figure (1.2): EC3



$$\bar{v}_1 = \bar{v}_2 + \bar{z}_{cc}\bar{\imath}_2 \qquad (1.17)$$

$$\bar{\imath}_1 = \bar{y}_0\bar{v}_2 + (\bar{y}_0\bar{z}_{cc} + 1)\,\bar{\imath}_2 \qquad (1.18)$$

Per-unit impedances

Application

Usually, the datasheet of a transformer contains at least the following information:

- Short circuit voltage $\boldsymbol{V_{cc}}$ % (in percent of the nominal voltage)
- Copper loss $\boldsymbol{P_{cc}}$ % (in percent of nominal power)
- Iron loss $\boldsymbol{P_0}$ % (in percent of nominal power)
- Open circuit current $\boldsymbol{i_0}$ % (in percent of the nominal current)

Thanks to the per-unit method is it possible to calculate the parameters directly by using the following equations:

Typical values:

$$\boldsymbol{z_{cc}} = Z_{cc_1}\frac{A_b}{V_{b_1}^2} = \frac{V_{b_1}V_{cc}\%}{\sqrt{3}I_{b_1}100}\frac{A_b}{V_{b_1}^2} = \frac{\boldsymbol{V_{cc}\%}}{\boldsymbol{100}} \qquad z_{cc} = 0.070 - 0.150 \text{ pu} \qquad (1.19)$$

$$\boldsymbol{r_{cc}} = R_{cc_1}\frac{A_b}{V_{b_1}^2} = \frac{A_b P_{cc}\%}{3I_{b_1}^2 100}\frac{A_b}{V_{b_1}^2} = \frac{\boldsymbol{P_{cc}\%}}{\boldsymbol{100}} \qquad r_{cc} = 0.002 - 0.005 \text{ pu} \qquad (1.20)$$

$$\boldsymbol{y_0} = Y_{0_1}\frac{V_{b_1}^2}{A_b} = \frac{\sqrt{3}I_{b_1}I_0\%}{V_{b_1}100}\frac{V_{b_1}^2}{A_b} = \frac{\boldsymbol{I_0\%}}{\boldsymbol{100}} \qquad y_0 = 0.005 - 0.025 \text{ pu} \qquad (1.21)$$

$$\boldsymbol{g_0} = G_{0_1}\frac{V_{b_1}^2}{A_b} = \frac{3A_b P_0\%}{3V_{b_1}^2 100}\frac{V_{b_1}^2}{A_b} = \frac{\boldsymbol{P_0\%}}{\boldsymbol{100}} \qquad g_0 = 0.001 - 0.002 \text{ pu} \qquad (1.22)$$

$$\boldsymbol{x_{cc}} = \sqrt{z_{cc}^2 - r_{cc}^2} \qquad \boldsymbol{b_0} = \sqrt{y_0^2 - g_0^2}$$